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## Monte Carlo Schedule Risk Analysis

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One of the fundamental questions of project management is, “What will be the duration and cost of the project given the multiple risks and uncertainties?” Program Evaluation and Review Technique (PERT) and Monte Carlo analysis may help to answer these and other questions. Monte Carlo analysis is a straightforward approach to deal with complex sets of project uncertainties. However, both Monte Carlo and PERT have a number of limitations that are related to the manner in which we identify and interpret uncertainties.

### **How much will it really cost?**

With the recent increase in oil prices, Calgary, the oil capital of Canada, has experienced tremendous growth. Population has reached one million people and the cost of all major infrastructure projects has risen dramatically. The original price tag to complete a section of Calgary’s ring road was pegged at \$250 million Canadian dollars in 2004. By the middle of 2006, cost overruns were estimated to be \$235 million (Braid, 2006). One third of this amount could be attributed to changes in the project scope, which included two new intersections; however, the remaining two-thirds was caused by increases in the cost of labor, material, and fuel. Due to increased costs, the project will be delayed for at least one year. Similar situations occurred with the construction of a causeway in the city’s south, where costs jumped from \$100 million to \$150 million, and for a wastewater plant, which experienced an increase from \$220 million to \$350 million. When cost overruns and delays of these proportions occur, two questions need to be asked:

*Why did it happen?*

*What do we do next?*

We are all blessed with remarkable hindsight; analyzing the past is always easier than predicting the future. In the examples above, city and provincial officials, the parties responsible for financing the projects, were trying to find a common strategy to deal with

the escalating costs. One option was to delay the projects; another was to continue with the original plan, as costs on the deferred projects will increase even more, but how much more? The answer to this question will drive the decision. And what will be the final tally for these projects given all the uncertainties that surround labor, fuel, and material costs during the oil boom in Calgary? For example, will \$235 million be the total cost overruns in the ring road project or will it be even more given the environment?

Nothing is certain in our world and this includes project durations, finish times, costs, and other parameters. Therefore, it is not possible to say that this road construction will cost exactly

The chance that project will be completed on time and within budget is one of the most important indicators for the decision-making.

\$250 million. What we can say is that that **there is a chance** that project will cost \$250 million.

But what is the nature of this chance? If we say that there is a 90% chance that the project will cost \$250 million or less, this implies that we are very confident that the project will be

completed within budget. However, if the chance is 20%, this means that we do not have a lot of confidence in the estimate and that we need either to review the project scope and resources, or accept cost overruns. By quantifying the chance for each project scenario, you can review different project alternatives and choose the one that has the highest chance of successful completion.

So we need to find the answers to two very important questions, which will help us to make our decision.

1. How much would the project cost and how long would it take given all the risks and uncertainties associated with the project?
2. What is the chance that project will be completed on time and within budget?

If we know the risks and uncertainties associated with activities within a project, we can perform calculations to find an answer to these questions. The simplest way to calculate the effect of risks and uncertainties is to create many schedules of the same project with combinations of input parameters: risks, different estimates of activity's cost and duration, resources, and so on. We can then analyze all these scenarios together to find the answer to these questions.

This method is called **scenario analysis**. It is a simple and straightforward approach you can use without any sophisticated tools. It works very well for simple projects or for a particular phase of a project. PMBOK® Guide recommends what-if scenario analysis as one of the **schedule network analysis** techniques.

The problem with this approach is that accurately representing the combinations of risks and uncertainties that exist in most projects will produce an unmanageably large number of scenarios. Each project has large number of tasks and resources, and each task and resource can have different risks and uncertainties. To further complicate the analysis, these risks can occur at different times, and we need to find the cumulative impact of these risks on the project. People have developed a number of quantitative methods that can help to overcome this problem.

## PERT

Very often major technological advances are by-products of military research. Between 1956 and 1958, the consulting firm Booz Allen Hamilton assisted the U.S. Navy's Special Project Office with the development of the Polaris Fleet Ballistic Missile program. This project was probably one of the largest and riskiest research and development efforts the US military had ever undertaken. Managers wanted estimates of the probabilities of meeting important milestones, such as test-launching a missile on a particular date. A by-product of this project was the **Program Evaluation and Review Technique** or **PERT** (Craven, 2001). Developed 50 years ago "Classic PERT" is well known today, although applications are limited. Here is how it works:

**Expected duration** (t) of the activity or mean is calculated using the following formula:

(Optimistic duration + most likely duration\* 4 + Pessimistic duration)

t = -----

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This formula allows managers to use expected durations to create project schedules. But why not just create an optimistic or pessimistic schedule using these durations? Here is a simple example that will help to explain why PERT uses expected durations instead of optimistic or pessimistic durations separately.

Let's imagine a construction project, which requires the installation of ten prefabricated columns. The installation of one column takes between 2 and 4 hours, and the columns are installed one after another. However, if there is a problem with the installation of one column, this does not mean that there will be a problem with the other columns. If one column can take 4 hours, is it rational to determine that the pessimistic duration for installing all of the columns will be 4 hours x 10 columns = 40 hours? Not really, this would be an **extremely pessimistic** duration, which could happen only if installation of all columns experienced the same problem, which is highly unlikely.

This example illustrates one of the biases associated with **anchoring and adjustment heuristics**. People tend to significantly overestimate or underestimate probability of conjunctive (where two conditions must exist) events. The four-hour duration is an anchor that can lead to an incorrect judgment.

As we can see, if we use optimistic (pessimistic) task durations to create optimistic (pessimistic) schedules, frequently we will get misleading results. Therefore PERT, by using the expected duration approach was a major step toward the incorporation of uncertainties in project management.

More importantly, PERT also included simple formulas and methods to calculate the **probability** of meeting specific milestones and therefore was easy for project managers and teams to use without any arduous training.

With all the elegance of PERT, it has a number of problems:

- Classic PERT gives accurate results only if there is a single dominant path through a precedence network. When a single path is not dominant, classic PERT usually provides overly optimistic results (Klastorin 2003).

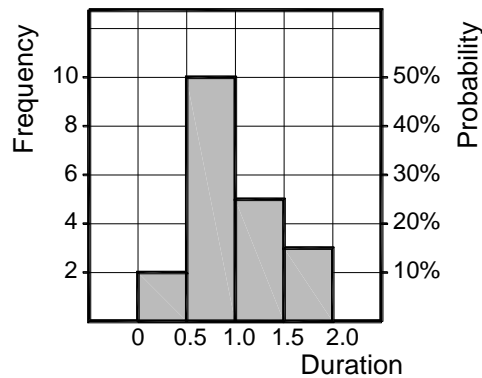


Figure 1. Frequency histogram

- According to PERT, uncertainties associated with tasks are independent of each other, which is not true in many cases.
- When we are trying to estimate the most optimistic, the most pessimistic, and the most likely duration of the task, we will be affected by the anchoring heuristic. The most likely duration will become an unwanted anchor, which will ‘skew’ our ability to accurately estimate task or project durations

To address these and other challenges, other analytical approaches have been developed. But before we can discuss them, we need to give a quick overview of some basic concepts that are used with these methods.

### Statistical Distributions

This may seem strange, but many people do not have a concept of maps. When they look at a map of a city, they cannot find a correlation between what they see on a map and what is physically around them. A similar situation can occur with statistical distributions. Many project managers are familiar with the concept of statistical distributions, perhaps from a basic course on probability and statistics, but they are unable to tell anybody (including themselves) how it actually works or its practical application. So, if you find yourself in this group, here is a quick primer.

Let’s assume that you are trying to analyze the duration of the activity “Install Kitchen Sink”. Depending on the type of sink, configuration of pipes, and other factors, it can take varying lengths of time. So after installing twenty sinks and recording the duration each time, you can develop a record of the task durations (Table 1).

Duration of the activity “Install Kitchen Sink”	Occurrences	Probability: Number of occurrences divided by total number of installations (20)
Between 0 and 0.5 hours	2	$2/20=0.1$ (10%)
Between 0.5 and 1 hours	10	50%
Between 1 hour and 1.5 hours	5	25%
Between 1.5 hours and 2 hours	3	15%

Table 1. Activity duration on different trials

You can actually represent this as a chart with **duration** on the horizontal axis and **frequency** on the vertical axis. The chart can also display the probability for certain durations (Figure 1).

A **statistical distribution** is an arrangement of values showing their frequency of occurrence. You may redraw this chart in another format. For each point of the chart you can add up all frequencies (probabilities) associated with all points on the left of the selected point. This is how we calculate cumulative probability and the chart is called a **cumulative probability plot**. These manipulations allow us to determine the probability associated with a certain value.

For example, what is the probability that the duration will be 1.2 hours? To find out, locate the duration 1.2 hours on the horizontal axis and draw a line up to solid line. Now draw a straight line to the right to find the probability, which in this example equals 85%. Unfortunately, cumulative probability plots can be confusing. They do not help us understand the interval for the parameter in question (in this case, task duration). It is only useful if we draw lines and get a probability as shown on Figure 2.

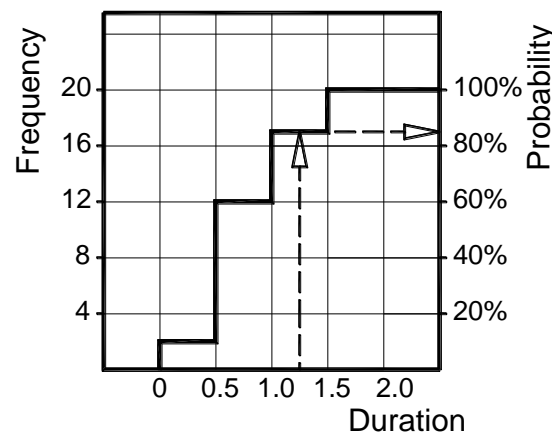


Figure 2. Cumulative probability plot

If you have an empirical dataset, you may create a very irregular or spiked distribution curve and you may want to make this distribution smoother using one of **continuous distributions**. Continuous distributions are defined by different mathematical formulas. Such continuous distributions are usually a better reflection of the nature of the real-life data because it has a continuum of possible outcomes. While there is a large list of different continuous distributions, only few of them are actively used in project management (Figure 3). Among them are:

**Uniform** – there is an equal probability that the parameter will be within certain range.

**Triangular** – the parameter is estimated using minimum, maximum, and most likely estimates; minimum and maximum are not optimistic and pessimistic estimates, they are extremes.

**Normal** – a symmetrical distribution, which occurs very often in business and in nature. Remember that this distribution is **unbounded**, which means that it spreads to infinity from both ends; in project management it needs to be used with some type of cutoff to remove the infinities.

**Lognormal** – a positively skewed (non-symmetrical) distribution (has longer tail to the right).

**Beta** – a bounded distribution, which uses a mathematical formula that includes two coefficients. By changing these coefficients, beta distribution can take variety of shapes; it can be symmetrical or non-symmetrical. The PERT formula was derived using a Beta distribution.

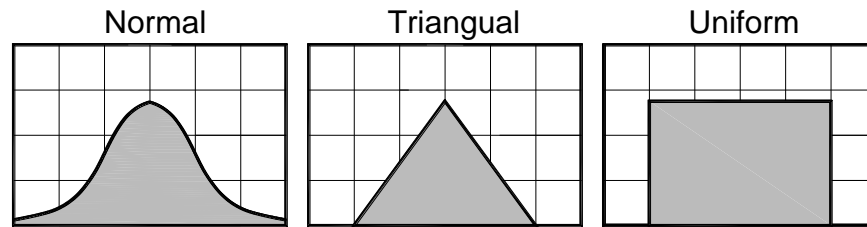


Figure 3. Different continuous statistical distributions

In addition to these distributions that are defined by recognizable mathematical formulas, most software tools will allow you to create **custom** distributions. All that is required is data, which in this case is the frequency of occurrence of a certain value. For example, you can enter the distribution shown on Figure 1 and use it for further analysis. Moreover, if you have empirical data, you can find known statistical distributions that can fit to this data. There are numerous software tools that will help you to choose the **best fit** among different types of distributions for your data.

When most people think about statistical distributions, the first thing that comes to their minds is a picture of a chart. But remember, statistical distributions are really only an arrangement of values and there are many parameters that can be used to analyze the distribution. Most important parameters are:

**Mean** –a mathematical average, calculated as the sum of variable values for all the trials divided by number of trials.

**Standard Deviation** - a measure of how widely dispersed the values are in a distribution. The greater the standard deviation, the more uncertainty is associated with the parameter.

**Percentile** - A value on a scale of zero to one hundred that indicates percentage of a distribution that is equal to or below this value. A value in the 95<sup>th</sup> percentile (sometimes defined as P95) is a value equal to or better than 95 percent of all other values in the distribution.

### **Monte Carlo Technique: How it Works**

Now that we have provided a little background on statistical distributions, we can explain how you can use Monte Carlo techniques to analyze a project schedule with uncertainties. Monte Carlo methods were originally practiced under more generic names such as "statistical sampling". Actually the name "Monte Carlo", popularized by early pioneers in the field including Stanislaw Marcin Ulam, Enrico Fermi, John von Neumann and Nicholas Metropolis, is a reference to the famous casino in Monaco. Its use of randomness and the repetitive nature of the process are analogous to the activities conducted at a casino. Stanislaw Marcin Ulam tells in his autobiography *Adventures of a Mathematician* that the method was named in honor of his uncle who was a gambler (Ulam, 1991).

Here is how it works. Let us assume that you have performed a calculation of some type. The type of calculation does not matter. For example, it can be an economic model defined in the spreadsheet. However, in this example, we are performing schedule network analysis using critical path method (CPM). Each project schedule has a number of uncertain (probabilistic) parameters: task duration, task cost, start and finish times of a task, rates associated with resources, and others. Our goal is to come up with statistical distribution for project cost and duration, which we develop by running simulations. Here are the steps of the simulation process:

1. Retrieve the values of the parameter from the statistical distribution. This process is called **sampling**. Basically, you roll a die and return a random number. You will use this number in a mathematical formula associated with the distribution. The formula will return a value. It is at this point that the magic of Monte Carlo occurs: if you roll a die many times, values associated with the hump or spike of the distribution will come up more often than the values associated with the “low-lying areas” (or tails) of the distribution.
2. Use these values in the calculation engine that you are using to run your model, which in this case is a project schedule. In other words, your project schedule will be calculated using a critical path method based on task duration, cost, and the other parameters we retrieved from the statistical distribution in the previous step.
3. Save the results of the analysis (project duration, cost, and other parameters). Repeat the process hundreds of times, each time using a new set of values from the statistical distribution. Each separate run is called a **trial**. After you have calculated and saved the results of hundreds of trials, you will now have a distribution of project parameters, which you can represent on a chart similar to those shown in Figure 1 and Figure 2.

Luckily, you don't have to perform all these calculations manually as there are many software tools specifically designed for this task. Your job is to define the distributions for input parameters and analyze the results. One of these software packages is RiskyProject by Intaver Institute Inc. ([www.intaver.com](http://www.intaver.com)).

Here is a more concrete example. Assume that you have a project with three tasks (Figure 4). The duration of the first task is defined by a normal distribution (with a mean of 4 days), the duration of the second task is defined by uniform distribution (between 3 and 7 days), and the duration of the third task is deterministic (always 4 days).

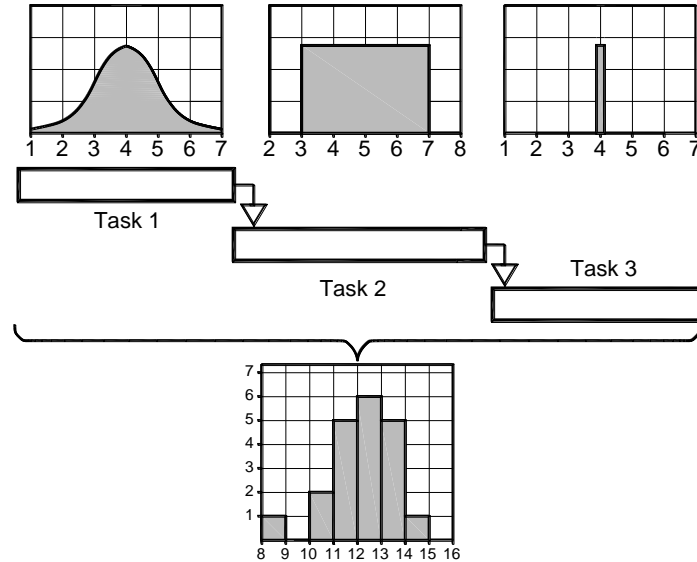


Figure 4. Monte Carlo simulation process

For this example we will only run twenty trials. We use a sampling to get the duration distributions for tasks 1 and 2. Take a look at Table 2. For task 1, we know that durations between 3 and 5 days occur more frequently than other durations because that is where the peak of the distribution curve or the hump is. For task 2, because it has a uniform distribution all durations are equally distributed between 3 and 7 days. For each trial, we add up all the task durations to get the project duration. Results will be displayed in a histogram. As you can see, the histogram for the project duration also has a hump due to the Task 1's normal distribution.

Using the data from the twenty trials, we can now calculate all of the probabilistic project parameters, including the mean (in this example it is 12.185 days), standard deviation, and different percentiles.

Trial	Task 1	Task 2	Task 3	Project	Trial	Task 1	Task 2	Task 3	Project
1	1.2	3.5	4.0	8.7	11	4.8	3.1	4.0	11.9
2	4.0	2.8	4.0	10.8	12	4.2	4.9	4.0	13.1
3	2.5	4.0	4.0	10.5	13	3.9	5.5	4.0	13.4
4	3.0	6.0	4.0	13.0	14	2.3	5.1	4.0	11.4
5	3.5	4.4	4.0	11.9	15	5.8	3.1	4.0	12.9
6	4.2	3.9	4.0	12.1	16	3.4	3.9	4.0	11.3
7	3.8	6.2	4.0	14.0	17	4.6	3.7	4.0	12.3
8	4.4	4.4	4.0	12.8	18	3.7	4.8	4.0	12.5
9	2.1	5.9	4.0	12.0	19	3.9	3.5	4.0	11.4
10	4.1	5.8	4.0	13.9	20	4.3	5.5	4.0	13.8

Table 2 Monte Carlo simulation results

### Which Distribution Should Be Used?

The distribution you choose for your project schedule may be based on an analysis of the historical data related to this parameter. For example, if the task occurred regularly each



time, you can measure the duration of the task and it can be used to define the distribution.

Unfortunately, historical data simply doesn't exist for many projects. Let us see how expert judgment may help define statistical distributions. The **probability method** (Goodwin and Wright, 2004) helps to mitigate negative effects of anchoring including insufficient adjustment:

1. Ask an expert to establish a range of values for the parameter.
2. Ask an expert to imagine a situation that could lead to value lying outside the range and revise the range if necessary.
3. Divide the range into 4-7 intervals and for each interval ask the expert to assess whether they could increase or decrease that value. For example, an expert estimated that duration range is between 5 and 10 days. Ask the expert, "What is the chance that duration is less than 6 days?", then "What is the chance that duration is less than 7 days?" and so on. By the end, you have elicited a cumulative probability distribution. You may draw the distribution, connect the points by hand and fit the statistical distribution.
4. Perform a reality check. First, you may ask an expert to come up with cumulative probabilities using different intervals, for example 1.5 days instead of 1 day. Then you can compare the results with the previous assessment and make necessary corrections if needed. You can also ask the experts to define where they think the peak of the distribution should be and compare that with the results found in step 3.

Another method of eliciting judgment for continuous distributions is the **method of relative heights**. Using the previous example, you can ask the experts how many times the duration will be between 5 and 6 days, between 6 and 7 days and so on. Then you can draw a frequency histogram, similar to the chart shown on Figure 1.

Unfortunately, if your project schedule has more than a few dozen tasks, it could take quite a while to come up with statistical distributions for all uncertainties. The good news is that with most project schedules, if you know the range of the data, the particular shape of the distribution may be not very critical for the analysis. If you choose a triangular distribution for particular task duration instead of lognormal, this will not completely skew up your analysis as long as your range is accurate. Risks and uncertainties that you have not accounted for will cause you more problems than an inaccurate distribution shape.

### **How many trials are required**

One of the first questions many people ask about the Monte Carlo process is how many trials are needed to perform a meaningful analysis. The answer depends upon the nature of the uncertainties associated with the project schedule. In some cases, you may want to incorporate very rare events that have dramatic outcomes into the project schedule. For example, there is 1 chance in 1000 that a natural disaster will strike. While this event can be modeled using a discrete distribution, you will need to run at least 1000 trials to see how the event could affect the project schedule. Apart from these special cases, it is our experience that you can perform a meaningful analysis on most project schedules,

including very large schedules with over a thousand activities, with only a couple of hundred trials.

Fortunately, most software applications used in this field have a feature called convergence monitoring. After each trial, the software will calculate statistical parameters (mean, standard deviation, and possibly others) of a selected project variable, e.g. project cost or duration. The software will calculate the relative difference between these statistical parameters on two consecutive trials and if this difference stays within a specified variance over a number of consecutive trials, the software will deem that the results have converged and halt the Monte Carlo process. For example, if the relative difference between project standard deviations is less than 0.5% on more than 25 consecutive trials, the process ends.

### **Analysis of Monte Carlo Results**

#### What is the chance that the project will be on time and within budget

Monte Carlo helps us answer this very important question. Technically, we need to record the number of trials in which the project was on time and within budget and divide this by the total number of trials. For example, if you ran 100 trials and in 65 of them the project was on time, the chance would be 65%. The best way to analyze this is to use a statistical distribution associated with the project duration. Fortunately, the software tools you are using should have an interactive histogram for the statistical distribution: enter the date and you will be able to view the chance that the project will be completed before or on a particular date.

#### Sensitivity and Correlations

Using Monte Carlo, you can identify a task's uncertainties that will have the greatest effect on the project schedule. For example, if a task is very risky, it can significantly affect the project duration. In addition, you can identify correlations between tasks and monitor how they affect the project schedule.

#### Critical Indices

If we analyze a deterministic project schedule, we can identify a critical path. However, when we use Monte Carlo, the critical path can be different in each trial. In this case, we can determine the percentage of time a task is on the critical path during the trials. For example, as a result of the analysis, we find that Task A will be on the critical path 60% of the time, Task B 30% of the time, and Task C 45% of the time. In this example, Task A would be the most critical task and needs to be examined further. These are called critical indices and are valuable in identifying critical tasks that have risks and uncertainties.

#### Probabilistic Calendars

If there is a storm on the coast, you cannot continue a seaport improvement project during this time. Using Monte Carlo analysis, you can define the chance that certain calendar with working or non-working days will be used. For example, a storm calendar could be used 5% of the time to take into account the effect of poor weather conditions.

#### Deadlines

If a task reaches a certain deadline, one of the results is that the project or task will be canceled. The issue is that we do not know exactly whether the project will reach the deadline and what is the chance that the deadline will be missed. Monte Carlo will help you to answer this question, as it is easy to count the number of trials in which the deadline is missed.

### Conditional branching

Let's assume that your project schedule includes two different branches representing two different alternatives. Conditional branching allows the project to branch from one task to another task based on certain conditions. For example, the task duration is 6 days +/- 2 days (Figure 5). If the task is completed within 6 days, one project scenario will be selected, but if it is completed after 6 days, the other alternative will be selected. These types of conditions can be based not only on duration, but also on finish time, cost, and other parameters.

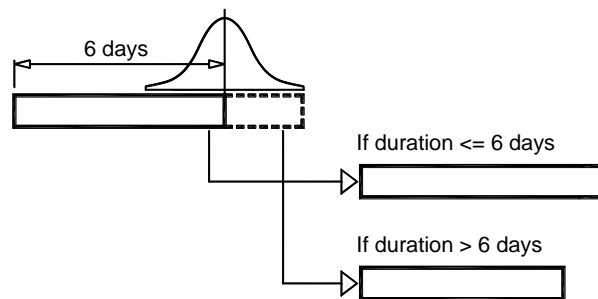


Figure 5. Conditional branching

### Probabilistic branching

Probabilistic branching allows the project to branch from one task to another task or group of tasks as part of the simulation process. For example, there is 50% chance that one branch will be selected and 50% chance that another branch will be selected.

### Chance of task existence

If you use probabilistic and conditional branching, some alternatives will be executed in one trial but not another. Therefore, you can count how many times the task was executed. This is the chance that the task will be executed or exists in a project.

### **Conclusion: Is Monte Carlo the Ultimate Solution?**

In essence, Monte Carlo allows a manager to model a huge number of combinations of project scenarios as part of one

PERT and then Monte Carlo represented major steps forward in project decision analysis. By understanding the historical trends for the costs of fuel, labor, and raw materials and accurately forecasting these costs into the future, we can **predict** the cost of projects. Based on these

forecasts, we can make decisions for complex portfolios such as the City of Calgary construction projects, which we discussed at the beginning of this paper. PERT and Monte Carlo methods will help us to determine what may happen to a project taking into account the cumulative effect of many risks and uncertainties.

However, for a number of reasons, Monte Carlo has not yet become part of the standard toolset used by project managers. Many project managers are unfamiliar or uncomfortable with the process even though it is covered in the PMBOK® Guide. Moreover, and perhaps more importantly, Monte Carlo and PERT have not been more widely adopted because project managers understand that inaccurate data will lead to inaccurate results (“Garbage in/Garbage Out” principle). Monte Carlo by itself will not solve the fundamental issues related to defining uncertainties:

1. Terry Williams (Williams, 2004) noted that project managers don’t sit and wait when a project slips. They perform certain **recovery actions**, which in most cases are not taken into account by Monte Carlo. In this respect, Monte Carlo may give overly pessimistic results. At the same time, as we already know, we are all subject to the **overconfidence** bias, which leads to overoptimistic project schedules. Unfortunately, the combination of optimistic and pessimistic results does not equal accurate results. Instead, you will get inadequate results.
2. Defining distributions is not a trivial process. Distributions are a very abstract concept that many of us find difficult to work with. To define distributions accurately, we have to perform a few mental steps that can be easily overlooked. Whether we are performing estimates of project parameters or developing distributions, we are affected by the cognitive and motivational biases.

As with everything else, Monte Carlo is not a panacea, but it is an excellent tool for the following situations:

- a. You have either reliable historical data or data that you can use to create a reliable probabilistic forecast. For example, you can predict the cost of raw materials within a certain range.
- b. You have tools to track actual data for each phase of the project and can perform Monte Carlo analysis at each phase to update your schedule.
- c. You have group of experts who understand the project, have experience in similar projects, and are trained to avoid cognitive and motivational biases when they define uncertainties and provide estimates.

If your project does not meet at least one of these criteria, Monte Carlo analysis may not substantially help to improve your decision-making. There are a lot of projects that do not meet these criteria. In particular, research and development projects fall into this category. Fortunately, there is a schedule network analysis technique, event chain methodology, which can help to address the shortcomings of Monte Carlo and PERT.

## Summary

- Project managers make decisions based on the answer to a fundamental question, “What will be the duration and cost of the project given its multiple risks and uncertainties?”
- PERT is an easy to use analytical method, but it has a number of major limitations.

- Monte Carlo analysis is a straightforward approach to deal with project uncertainties. Monte Carlo can be used effectively if accurate historical data is available, project tracking is performed, and trained project-specific experts are involved.

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